CORE MATHEMATICS (C) UNIT 1 TEST PAPER 8

1. Express in its simplest form without brackets: $(x-2y)^2(x^2-4y^2)$ [4]

2. Find the coordinates of the minimum point on the graph of $y = 8x + \frac{1}{2x^2}$. [5]

3. The lines l_1 and l_2 have equations $3y = 2x - 4$ and $2y = 3 - 3x$.

   (i) Find the coordinates of the point where l_1 and l_2 intersect. [4]

   (ii) Show, by calculation, that l_1 and l_2 are perpendicular. [2]

4. (i) Find the exact value of $\left(\sqrt{2}+\sqrt{18}\right)^2$. [3]

   (ii) Express in its simplest form with a rational denominator: $\frac{1}{5-\sqrt{15}}$. [3]

5. The line $y + x = 23$ cuts the curve $y = 25 - (x-4)^2$ at the points P and Q.

   The x-coordinate of P is less than that of Q.

   Find the coordinates of P and of Q. [6]

6. The line with equation $x + y = k$ meets the circle with equation $x^2 + y^2 = k$ in two distinct points. Find the range of possible values of k. [7]

7. The equation of a curve is $y = x^3 - 5x^2 + 4x + 2$.

   (i) Find an equation of the tangent to the curve at the point $(2, -2)$. [4]

   (ii) Find the x-coordinates of the points on the curve where the tangent has gradient -3. [4]

8. Sketch graphs of each of the following, showing clearly the behaviour of the graphs as they intersect or approach the coordinate axes.

   (i) $y = (x + 1)^2(x - 1)$, [3]

   (ii) $y = -2\sqrt{x}$, $x > 0$, [3]

   (iii) $y = -\frac{1}{x^2}$, $x \neq 0$. [3]

9. Express in the form 9^y

(i) $\frac{1}{9^{1-x}}$, (ii) 81^{x-2}, (iii) 3^{4x+6}. [5]

Hence, or otherwise, find the value of x for which $\frac{1}{9^{1-x}} = \frac{81^{x-2}}{3^{4x+6}}$. [4]

10. The circle C has centre $(-2, t)$ and radius $2\sqrt{2}$.

(i) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where b and c are to be expressed in terms of t. [3]

Given that C passes through the point $P\ (0, 2)$,

(ii) find the possible values of t. [3]

Given also that $t > 0$,

(iii) find an equation of the tangent to C at P. [4]

(iv) Find the area of the triangle formed by the tangent at P, the x-axis and the y-axis. [2]

CORE MATHS 1 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1. $(x^2 + 4y^2 - 4xy)(x^2 - 4y^2) = x^4 - 4x^3y + 16xy^3 - 16y^4$ — M1 A1 M1 A1 — 4

2. $\frac{dy}{dx} = 8 - \frac{1}{x^3} = 0$ when $x = 1/2$ Point is (1/2, 6) — M1 A1 M1 A1 A1 — 5

3. (i) $2x - 3y = 4$, $3x + 2y = 3$ $6x - 9y = 12$, $6x + 4y = 6$ — M1
   $13y = -6$ $y = -6/13$ Point is (17/13, –6/13) — M1 A1 A1
   (ii) Gradients are 2/3 and –3/2 Product = –1, so perpendicular — M1 A1 — 6

4. (i) $2 + 18 + 2\sqrt{2}\sqrt{18} = 20 + 2(6) = 32$ — M1 A1 A1
   (ii) $\frac{1}{5-\sqrt{15}} = \frac{(5+\sqrt{15})}{(5-\sqrt{15})(5+\sqrt{15})} = \frac{5+\sqrt{15}}{10}$ — M1 A1 A1 — 6

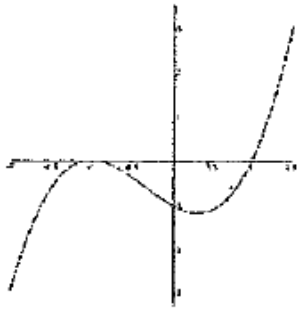
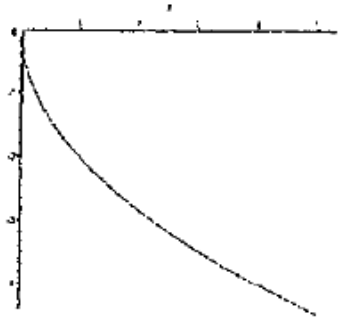
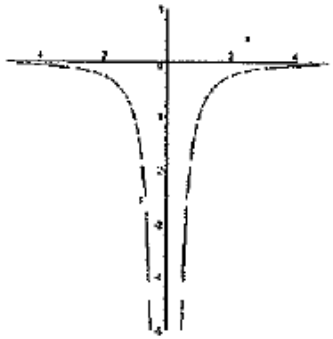
5. $23 - x = 9 + 8x - x^2$ $x^2 - 9x + 14 = 0$ $x = 2, x = 7$ — M1 A1 M1 A1
   P is (2, 21), Q is (7, 16) — A1 A1 — 6

6. $x^2 + (k - x)^2 = k$ $2x^2 - 2kx + (k^2 - k) = 0$ — M1 A1 A1
   For 2 real roots, $4k^2 - 8(k^2 - k) > 0$ $4k(k - 2) < 0$ $0 < k < 2$ — M1 A1 M1 A1 — 7

7. (i) $dy/dx = 3x^2 - 10x + 4 = -4$ when $x = 2$ $y + 2 = -4(x - 2)$ — M1 A1 M1 A1

(ii) When $dy/dx = -3$, $3x^2 - 10x + 7 = 0$ $(3x - 7)(x - 1) = 0$ B1 M1

$x = 1$ or $x = 7/3$ A1 A1 8

8. (i)  (ii)  (iii) 

B3 B3 B3 9

9. (i) 9^{x-1} (ii) $(9^2)^{x-2} = 9^{2x-4}$ (iii) $(9^{1/2})^{4x+6} = 9^{2x+3}$ B1 M1 A1 M1 A1

$x - 1 = 2x - 4 - (2x + 3)$ $x - 1 = -7$ $x = -6$ M1 A1 A1 A1 9

10. (i) $(x + 2)^2 + (y - t)^2 = 8$ $x^2 + y^2 + 4x - 2ty + (t^2 - 4) = 0$ M1 A1 A1

(ii) $4 - 4t + t^2 - 4 = 0$ $t = 0$ or $t = 4$ M1 A1 A1

(iii) When $t = 4$, gradient of radius $= -1$ so gradient of tangent $= 1$ M1 A1

Tangent is $y = x + 2$ M1 A1

(iv) Tangent cuts axes at $(-2, 0)$, $(0, 2)$ so area of triangle $= 2$ units^2 M1 A1 12